

# A *K*-band central disc surface brightness correlation with scalelength for early-type disc galaxies, and the inclination correction

Alister W. Graham<sup>★</sup>

*Instituto de Astrofísica de Canarias, La Laguna, E-38200, Tenerife, Spain*

Accepted 2001 April 5. Received 2001 February 26; in original form 2000 October 31

## ABSTRACT

The *K*-band light profiles from two statistically complete, diameter-limited samples of disc galaxies have been simultaneously modelled with a seeing-convolved Sérsic  $r^{1/n}$  bulge and a seeing-convolved exponential disc. This has enabled an accurate separation of the bulge and disc light, and hence an estimate of the central disc surface brightness  $\mu_{0,K}$  and the disc scalelength  $h$ . There exists a bright envelope of galaxy discs in the  $\mu_{0,K}$ – $\log h$  diagram; for the early-type ( $\leq$ Sbc–Sc) disc galaxies  $\mu_{0,K}$  is shown to increase with  $\log h$ , with a slope of  $\sim 2$  and a correlation coefficient equal to 0.75. This relation exists over a range of disc scalelengths from 0.5 to 10 kpc ( $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ). In general, galaxy types Scd or later are observed to deviate from this relation; they have fainter surface brightnesses for a given scalelength.

With a subsample of 59 low-inclination ( $i \leq 50^\circ$ ) and 29 high-inclination ( $i \geq 50^\circ$ ) galaxies having morphological types ranging from S0 to Sc, the need for an inclination correction to the *K*-band disc surface brightness is demonstrated. Certain selection criteria biases which have troubled previous surface brightness inclination tests (for example, whether the galaxies are selected from a magnitude- or diameter-limited sample) do not operate in the  $\mu_{0,K}$ – $\log h$  diagram. Measured central disc surface brightnesses are found to be significantly ( $>5\sigma$ ) brighter for the high-inclination disc galaxies than for the low-inclination disc galaxies. With no surface brightness inclination correction or allowance for the trend between  $\mu_{0,K}$  and  $\log h$ , the standard deviation to the distribution of  $\mu_{0,K}$  values is  $\sim 1 \text{ mag arcsec}^{-2}$ , while the standard deviation about the mean  $\mu_{0,K}$ – $\log h$  relation decreases from  $0.69 \text{ mag arcsec}^{-2}$ , when no inclination correction is applied, to  $0.47 \text{ mag arcsec}^{-2}$  when the inclination correction is applied. Possible changes to the disc scalelength with inclination, as a result of radial gradients in the disc opacity, have been explored. The maximum possible sizes for such corrections are too small to provide a valid explanation for the difference between the low- and high-inclination disc galaxies in the  $\mu_{0,K}$ – $\log h$  diagram.

**Key words:** dust, extinction – galaxies: fundamental parameters – galaxies: photometry – galaxies: spiral – galaxies: structure – infrared: galaxies.

## 1 INTRODUCTION

Over the last three decades, the validity of Freeman’s law – the observation that the *B*-band central disc surface brightness of spiral galaxies is roughly constant (Freeman 1970) – has been investigated many times. The interest generated by this work is not surprising given its implications for the formation and nature of disc galaxies. For instance, if the central  $M/L_B$  ratio is constant in spiral galaxies, it would imply a constant central mass surface density which would provide a strong constraint on any potential

disc galaxy formation mechanism. Subsequent studies observed the scatter in Freeman’s law to increase, notably from the inclusion of fainter late-type ( $>$ Sc) spiral galaxies (van der Kruit 1987), vindicating the argument by Disney (1976, 1980) that Freeman’s law was an artefact of selection effects. This result was expanded upon by de Jong (1996b), who argued that Freeman’s law must be redefined as de Jong found only an upper limit to the central disc surface brightness term. Similarly, studies of low surface brightness (LSB) galaxies reveal a class of spiral galaxies with central disc surface brightnesses lower than the canonical Freeman value (Longmore et al. 1982; Bothun et al. 1987; Davies Phillips & Disney 1988; van der Hulst et al. 1993; McGaugh & Bothun 1994;

<sup>★</sup>E-mail: agramah@ll.iac.es

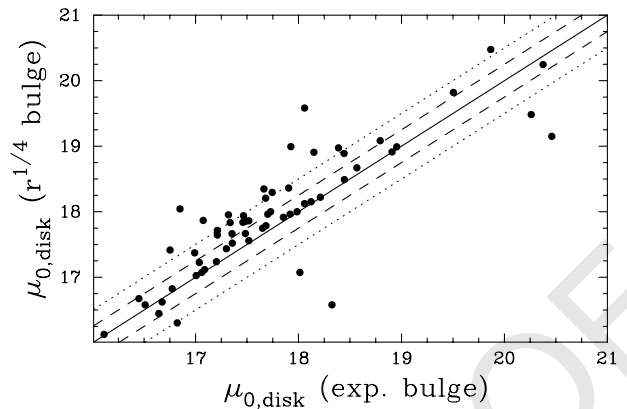
de Blok, van der Hulst & Bothun 1995; de Blok, van der Hulst & McGaugh 1996; McGaugh 1996; Bothun, Impey & McGaugh 1997) and with no indication of a lower limit.

An important consideration when measuring the central surface brightness of a disc is the question of the opacity of the disc. Extinction due to dust is an order of magnitude less in the  $K$ -band than in the  $B$ -band. The  $K$ -band also has the advantage of tracing the old stellar population, and not the few per cent (by mass) of young stars which dominate the light in the  $B$ -band. However, at least in the  $H$ -band, dust can still have a measurable effect on the disc surface brightness distribution (Aoki et al. 1991; Barnaby & Thronson 1992; Moriondo, Giovanelli & Haynes 1998). Amongst other things, this paper attempts to answer the following question: what inclination correction, if any, needs to be applied to the observed  $K$ -band surface brightnesses (and magnitudes) of disc galaxies? Applying the right inclination correction is important as it influences statistical studies of the properties of spiral galaxies. It affects derivations of the luminosity function (Leroy & Portilla 1998), and hence galaxy magnitude and redshift number counts, and it also affects corrections to the Tully–Fisher relation for studies of peculiar velocity flows and large-scale structure.

One of the difficulties in using the central surface brightness of a galaxy is that it is a combination of both disc and bulge light. For a galaxy sample that includes Sa to Sd–Sm galaxies, the contribution of the central bulge light to the central disc light can span a large range of values. Even for the Sc galaxies the bulge can contribute almost no extra light at the galaxy centre or it can be several magnitudes brighter than the disc. Kormendy (1977), Philipps & Disney (1983), and later Davies (1990) argued that failure to accurately model the contribution of bulge light may place constraints on the true range of central disc surface brightness values which are measured. Although the physical applicability of the parameter space explored in the models of these authors has been questioned (de Jong 1996b), at some level, the issue of bulge contamination remains.

Many of the pioneering papers which fitted both a bulge and disc generally modelled the bulge with a de Vaucouleurs (1948)  $r^{1/4}$  profile (Kormendy 1977; Burstein 1979; Boroson 1981, 1983; Kent 1985; Schombert & Bothun 1987). The risks of this assumption were mentioned by these authors and departures noted. de Jong (1996a) actually rejected the  $r^{1/4}$  bulge profile in favour of an exponential bulge profile, which, in general, gave a statistically better fit to his sample of 86 disc galaxies. Today, however, the range of bulge profile shapes has been investigated (Andredakis, Peletier & Balcells 1995, hereafter APB95; Moriondo, Giovanardi & Hunt 1998), and application of the classical exponential profile or  $r^{1/4}$  law to a bulge, which is better described by some other light profile shape, is known to introduce errors into the structural parameters of the galaxies (Graham & Prieto 1999; Graham 2001). While the effect is most dramatic on the bulge parameters, differences to the estimated central disc surface brightness can vary by  $0.5 \text{ mag arcsec}^{-2}$  or more, depending on the assumed bulge profile shape. To illustrate this, Fig. 1 shows the  $K$ -band central disc surface brightness estimates of de Jong (1996a), obtained when modelling the bulge with an  $r^{1/4}$  profile first and then with an exponential profile. One can see that considerable differences can arise when force-fitting these two restrictive classical bulge models that have no physical basis. In an effort to separate the bulge light from the disc profile as accurately as possible, a seeing-convolved  $r^{1/n}$  bulge profile has been simultaneously fitted with a seeing-convolved exponential disc profile in this analysis.

With a sample of both low- and high-inclination galaxies, the



**Figure 1.** The central  $K$ -band disc surface brightness,  $\mu_{0,\text{disk}}$ , derived by simultaneously fitting an  $r^{1/4}$  bulge with an exponential disc, and an exponential bulge with an exponential disc are plotted against each other (fitting and data from de Jong 1996a). The dashed and dotted lines mark the  $0.25$  and  $0.50 \text{ mag arcsec}^{-2}$  deviations from agreement.

selection of which is presented in Section 2, this paper identifies a considerable source of scatter to the  $K$ -band central disc surface brightness – namely, the dependence on the disc scalelength (preliminary results have already been presented in Graham 2000a,b). Section 3 provides a description of the standard inclination correction while Section 4 attempts to provide a quantitative measure to the size of the  $K$ -band surface brightness inclination correction needed for spiral galaxy discs. In Section 5 the consequences of an opacity dependence on radius, such that discs are more optically thick towards the centre, are explored and found not to be important in the  $\mu_{0,K}$ – $\log h$  diagram. Section 6 provides a discussion of the results and Section 7 presents the main conclusions from this investigation.

## 2 DATA

The data used in this investigation are from two statistically complete, diameter-limited samples of galaxies selected from the Uppsala General Catalogue of Galaxies (UGC; Nilson 1973). A selection of galaxies with inclinations less than  $50^\circ$  was investigated by de Jong & van der Kruit (1994) while a somewhat complementary sample of galaxies with inclinations greater than  $50^\circ$  was investigated by Balcells & Peletier (1994) and later APB95. Both studies avoided disturbed or interacting systems.  $K$ -band images taken with United Kingdom Infrared Telescopes (UKIRTs) IRCAM2 resulted in a volume-limited sample of 86 low-inclination spiral galaxies, for which the data reduction is described in de Jong & van der Kruit (1994).<sup>1</sup> Images taken with UKIRTs IRCAM3 resulted in a sample of 30 early-type ( $T < 5$ ) high-inclination spiral galaxies, for which the data reduction procedures can be found in APB95 and Peletier & Balcells (1997).<sup>2</sup> Owing to the complementary inclination selection criteria, no

<sup>1</sup>The surface brightness profiles for the low-inclination sample, obtained from the average surface brightness on ellipses of constant ellipticity and position angle, can be found at <http://cdsweb.u-strasbg.fr/hbini/Cat?J/A+AS/118/557>.

<sup>2</sup>The surface brightness profiles for the high-inclination sample can be found at <http://www.elsevier.nl/gej-ng/10/33/29/22/23/23/article.html>. (The  $K$ -band light profiles were taken from the ellipse fits, and not from the wedge profiles.)

common galaxies have been observed – preventing a check of the photometric zero point. This is unfortunate; however, although calibration offsets of 0.05 mag are not uncommon between different data sets, this would not have any significant impact on the results of this investigation and is certainly tolerable.

There is a difference between the diameter-limited selection criteria of each sample. The low-inclination galaxies were selected to have a red major-axis diameter  $\geq 2.0$  arcmin, while the high-inclination galaxies were selected to have a blue major-axis diameter  $> 2.0$  arcmin. The difference to the final high-inclination sample, had the selection criteria for the larger low-inclination sample been used, would have been small. Only one galaxy (UGC 9016) with a blue major-axis diameter greater than 2.0 arcmin and having a red major-axis diameter less than 2.0 arcmin was included in the final sample of 30 galaxies by APB95 – this galaxy is easily excluded from this analysis. Two galaxies (UGC 8485 and 10713) with a red major-axis diameter  $\geq 2.0$  arcmin but a blue major-axis  $\leq 2.0$  arcmin would have been included if the diameter selection criteria were the same.

APB95 applied further selection criteria which de Jong & van der Kruit (1994) did not, but which can be applied here to make the samples as similar to each other as possible. While de Jong & van der Kruit (1994) selected disc galaxy types from S0 to Irr, APB95 restricted their sample to galaxy types earlier than Sc. APB95 also applied a magnitude selection criterion, such that only galaxies with  $B_T < 14.0$  mag were included, and they excluded galaxies with bars. These criteria are subsequently applied both individually and all at once to the low-inclination sample to see if this affects the results of this investigation. The only major concern is that APB95 excluded three galaxies solely because they were observed to be very dusty, in the optical, all the way to the centre. This may well have an effect on the mean surface brightness of the high-inclination sample. However, this still left a sample of 30 high-inclination galaxies and so its impact is not expected to be terribly substantial but should, nevertheless, be noted.

Using the same algorithm, all galaxies have been simultaneously modelled with a seeing-convolved Sérsic (1968) bulge profile and a seeing-convolved exponential disc. The model parameters obtained are the bulge effective half-light radius ( $r_e$ ), the bulge surface brightness at this radius ( $\mu_e$ ), the shape of the bulge profile ( $n$ ), the central disc surface brightness ( $\mu_0$ ), and the disc scalelength ( $h$ ). The disc scalelength is that appropriate to the major-axis, and is not the geometric mean between the major- and minor-axis – a quantity shortened through inclination. A presentation of the fits to the low-inclination galaxies can be found in Graham (2001). Three of these galaxies have optical but no  $K$ -band data, and five were imaged under non-photometric conditions. Of the 86 low-inclination discs, 65 have morphological type  $T \leq 5$  (42 have  $T \leq 4$ ) – taken from the RC3 (de Vaucouleurs et al. 1991). Only two galaxies having  $T \leq 5$  could not be reliably modelled. These are UGC 8279 which has no apparent bulge – causing the bulge/disc decomposition to fail – and UGC 6028 whose disc has two distinct slopes. This left a sample of 59 low-inclination disc galaxies with morphological type  $T \leq 5$  (39 with  $T \leq 4$ ). For the sample of 59 galaxies, a  $V/V_{\max}$  test gave an average value of 0.53 – in agreement with the expected value of  $0.5 \pm 1/\sqrt{12 \times N}$  for a statistically complete, diameter-limited sample ( $= 0.5 \pm 0.04$  when  $N = 59$ ). Similarly, the smaller sample of 39 galaxies represents a statistically complete, volume-limited sample of  $T \leq 4$  disc galaxies, with the average  $V/V_{\max} = 0.49$ . Although the remaining later-type spiral galaxies are included in later figures – to help highlight that the selection criteria are not

clearly responsible for the observed trends – these galaxies have not been used in the statistical analysis which follows. All high-inclination galaxies, excluding UGC 9016, have been included. For these, the  $V/V_{\max}$  test gave an average value of 0.51 – within the range  $0.5 \pm 0.05$  expected for a sample of 29 galaxies that is randomly distributed in space.

The central surface brightnesses have been uniformly corrected for (i) Galactic extinction using the composite *IRAS* and *COBE/DIRBE* dust extinction maps of Schlegel, Finkbeiner & Davis (1998) – data obtained from NASA/IPAC Extragalactic Database (NED) (ii)  $(1+z)^4$  cosmological redshift dimming and (iii)  $K$ -correction using heliocentric velocities (Poggianti 1997). For the low-inclination galaxies, the average size of these corrections was relatively small at  $-0.02$ ,  $-0.05$ , and  $0.02$  mag respectively, with maximum corrections of  $-0.06$ ,  $-0.12$ , and  $0.04$  mag. For the high-inclination sample, these corrections were again small, with average values of  $-0.02$ ,  $-0.03$ , and  $0.01$ , and they do not significantly alter the results of this paper.

In this paper a value of  $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$  has been used. For both the low- and high-inclination galaxy samples, distances were derived using the heliocentric velocities from NED which were then corrected for the Virgo-infall using the 220 model of Kraan-Korteweg (1986).

### 3 THE STANDARD INCLINATION CORRECTION

In the case of a transparent disc, stellar light is radiated equally in all directions. Consequently, the viewing angle (inclination) will not affect the observed magnitude of the galaxy. However, transparent discs that are seen at higher inclinations (with inclination  $i$  increasing from face-on to edge-on) project to a smaller area on the sky and consequently their surface brightness (intensity per unit area) will appear brighter than if they had been viewed face-on. In contrast to this, for an optically thick disc the observed surface brightness will be independent of the viewing angle, as the line-of-sight depth into the disc will be the same for all inclinations and the observed intensity per unit area will remain fixed. For optically thick discs viewed at higher inclinations, the total galaxy magnitude decreases as the projected area of the galaxy is decreased from its face-on value.

This situation has been expressed quantitatively with a simple model such that

$$\mu_{\text{inc}} = \mu_{\text{face-on}} + 2.5C \log(b/a) \quad (1)$$

where  $b/a$  is the ratio of the semiminor to semimajor axis of the disc. For an optically thick disc  $C = 0$ , and for an optically thin disc  $C = 1$ . Of course, spiral discs may well be described by some intermediate degree of opacity, where  $C$  would take on a value somewhere between 0 and 1. The value of  $C$  is also known to be a function of wavelength since its origin is dust.

One can envisage a number of special locations for dust within a disc that would complicate the opacity test. In the text that follows, I shall refer to a disc that behaves transparently in an inclination test as a ‘transparent disc’. The disc may in fact not be transparent, or may be only semi transparent. In one case of interest, dust may reside in the disc with a scaleheight less than that of the stars. If so, as noted by Disney, Davies & Phillips (1989), the outer parts of the disc (in the  $z$ -direction) may be transparent, while obscuring dust is sandwiched between these two outer layers. As the inclination increases, the outer layers, as opposed to the disc as a whole, will

cause the observed surface brightness to brighten above its face-on value (Davies et al. 1993; Davies, Jones & Trewheila 1995). Another case of interest is if a dust ring was to encircle the disc. Although unlikely, this may occur to a degree by dust in the outer spiral arms (Huizinga 1994). For face-on galaxies and galaxies at intermediate inclinations, this would simply introduce a wiggle into the outer light profile, but for those galaxies which are very edge-on this would create a biased impression of the global opacity of the disc.

Irrespective of the location or distribution of dust, all future references to the value of  $C$  are made only with regard to the required, statistically averaged, inclination correction, and not with regard to the nature of the disc as a whole.

## 4 APPLICATION OF THE CORRECTION

### 4.1 A constant central disc surface brightness – Freeman’s law

The inclination correction was uniquely determined for each galaxy depending on the ellipticity of its disc. With  $C = 0$ , the inclination correction to the surface brightness term is zero. With  $C = 1$ , the average size of the inclination correction for the low-inclination sample of spiral galaxies is  $0.26 \text{ mag arcsec}^{-2}$ , and the maximum correction is  $0.58 \text{ mag arcsec}^{-2}$ . For the high-inclination sample of galaxies, when  $C = 1$ , the average correction is  $1.34 \text{ mag arcsec}^{-2}$  and the maximum correction is  $1.92 \text{ mag arcsec}^{-2}$  for a galaxy with a disc ellipticity ( $\epsilon = 1 - b/a$ ) equal to 0.83. In passing, it is noted that if some of the galaxy discs do not possess circular symmetry but are intrinsically elongated, then this will introduce a source of scatter into the inclination

correction. One may therefore be able to further reduce the scatter in the central disc surface brightness distribution by using kinematic, rather than photometric, inclinations (Andersen et al. 2000). Although, such inclination differences are greatest at small inclinations where the size of the inclination corrections are small.

The scatter in the distribution of central disc surface brightnesses is known to be larger in the  $K$ -band than in the  $B$ -band, and  $\mu_0$  is known to increase for galaxy types  $T > 5$  or 6 (Grosbøl 1985; Valentijn 1990; Peletier et al. 1994; de Jong 1996b; Tully & Verheijen 1997; de Grijs 1998) and this is also seen with the current sample. In Fig. 2, when  $C = 1$ ,  $\mu_{0,K}$  displays no trend with morphological type  $T \leq 5$ . Given the similarity in central disc surface brightnesses between the  $T = 5$  galaxies and the earlier types, they have been included in the statistical analysis which follows so as to increase galaxy numbers and improve the statistics. In later comparison studies between the low- and high-inclination samples, the effect of removing the  $T = 5$  galaxies, which are not present amongst the high-inclination sample, is explored.

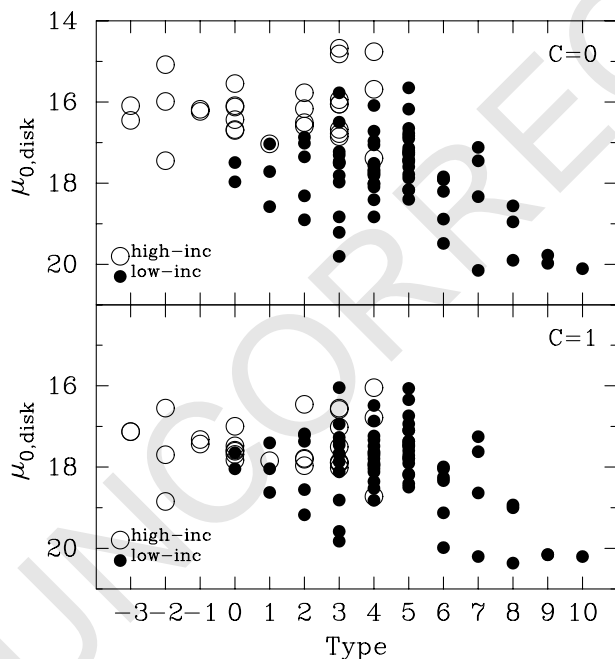
For the low-inclination galaxies with type  $T \leq 5$ , the mean value of  $\mu_{0,K}$  is  $17.45 \pm 0.76$  ( $C = 0$ ) and  $17.72 \pm 0.74$  ( $C = 1$ ). For the high-inclination sample, the mean value of  $\mu_{0,K}$  is  $16.10 \pm 0.70$  ( $C = 0$ ) and  $17.45 \pm 0.65$  ( $C = 1$ ). For the combined sample of galaxies with  $T \leq 5$ , the mean central disc surface brightness is  $17.01 \pm 0.98$  ( $C = 0$ ) and  $17.63 \pm 0.72$  ( $C = 1$ ). The inclusion of galaxy types with  $T > 5$  increases both the mean value of  $\mu_{0,K}$  and the associated error.

### 4.2 A correlation of central disc surface brightness with scalelength

A plot of  $\mu_{0,K}$  against  $\log h$ , where  $h$  is the scalelength of the disc, reveals – for the early-type disc galaxies – a correlation between these two parameters (Fig. 3). One can also see, when no inclination correction is applied ( $C = 0$ ), that for a given size (scalelength) of a disc, the high-inclination sample is observed to have brighter values of  $\mu_{0,K}$  than the low-inclination sample – indicating the transparent behaviour of at least the early-type discs. The decision to include the  $T = 5$  galaxies in the low-inclination sample does not appear to affect the observed  $\mu_{0,K} - \log h$  relation, whereas the later-type galaxies do show a departure from the relation defined by the early-type spiral galaxies. A discussion on the influence of the selection criteria on this apparent correlation is deferred to Section 6.

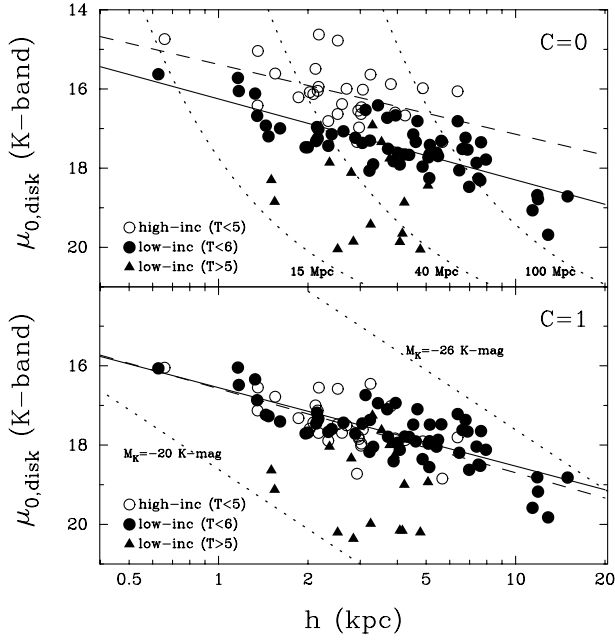
After application of the  $C = 1$  inclination correction, a Pearson correlation coefficient of 0.75 exists between  $\mu_{0,K}$  and  $\log h$  (kpc) for the combined sample of low- and high-inclination early-type ( $T \leq 5$ ) disc galaxies. The previous standard deviations in the distribution of  $\mu_{0,K}$  are 42 per cent and 53 per cent higher than the standard deviations about the mean  $\mu_{0,K} - \log h$  relation – which are  $0.69 \text{ mag arcsec}^{-2}$  (when  $C = 0$ ) and  $0.47 \text{ mag arcsec}^{-2}$  (when  $C = 1$ ).

From Fig. 3, it is clear that when no correction is made for inclination ( $C = 0$ ) the low- and high-inclination data sets have different mean central surface brightnesses. Subtracting the mean  $\mu_{0,K} - \log h$  relation, one can remove the dependence of surface brightness on galaxy size before computing the difference in mean values of  $\mu_{0,K}$  for the two samples. A Student’s  $t$ -test on the two residual data sets (low-inclination and high-inclination) reveals significantly different means. Using the F-test for the similarity of variances, these residual data sets have different variances at the  $1.5\sigma$  significance level. The Student’s  $t$ -test was therefore applied twice: assuming equal and then unequal variances. The probability



**Figure 2.** The central  $K$ -band disc surface brightness is plotted against galaxy morphological type, with no inclination correction ( $C = 0$ ) and the maximum inclination correction ( $C = 1$ ) applied. (The value of  $C$  comes from the inclination correction  $2.5C \log(b/a)$  given in the text.) The high-inclination ( $i > 50^\circ$ ) disc galaxy sample are represented with the open circles, and the low-inclination ( $i < 50^\circ$ ) disc galaxy sample are represented by the filled circles.





**Figure 3.** The central  $K$ -band disc surface brightness is plotted against the scalelength of the disc for the case when  $C = 0$  (i.e. no inclination correction) and when  $C = 1$ . The solid line is the linear regression of  $\mu_{0,K}$  on  $\log h$  for the low-inclination galaxies with type  $T \leq 5$ . The dashed line is the regression for the high-inclination galaxy sample. The dotted curves in the upper panel indicate the 2.0-arcmin diameter (1.0-arcmin radius) selection limit at the  $24.7 R$ -mag arcsec $^{-2}$  isophote for galaxies closer than 15, 40 and 100 Mpc, and assuming galaxies have perfect exponential discs with  $R - K = 2.25$ . The dotted lines in the lower panel indicate where discs of a fixed luminosity reside. A Hubble constant of  $75 \text{ km s}^{-1} \text{ Mpc}^{-1}$  has been used.

that the value of  $t$  from the Student's test could be as large as it is just by chance is smaller than  $10^{-7}$  – indicating that the difference in means is highly significant. In addition to subtracting the mean  $\mu_{0,K}$ – $\log h$  relation for the combined sample, the mean  $\mu_{0,K}$ – $\log h$  relation for the low-inclination sample, and also for the high-inclination sample, were separately subtracted and the Student's  $t$ -test was applied again. From all six  $t$ -tests (equal and unequal variances, and using the three different mean  $\mu_{0,K}$ – $\log h$  relations) the probability that the difference in means could be as large as it remained less than  $10^{-7}$ .

With  $C = 1$ , the same tests were again applied. This time, the variances of the two residual data sets were not only found to be equal at the  $1\sigma$  level, but were notably reduced for the high-inclination sample. The mean values of  $\mu_{0,K}$  from the residual data sets – having subtracted the mean  $\mu_{0,K}$ – $\log h$  relation – were found to be consistent with each other at well within the  $1\sigma$  confidence limit. That is, application of the Student's  $t$ -test reveals that with  $C = 1$ , the mean surface brightness offset between the low- and high-inclination galaxies in the  $\mu_{0,K}$ – $\log h$  plane is not significant.

To compute the mean offset, a modified application of the Working–Hotelling (1929) confidence bands was used (Feigelson & Babu 1992). This method of analysis performs a linear regression on one data set (the calibration set) and then, given the  $x$ -values ( $\log h$ ) of the second data set, predicts their  $y$ -values (in our case  $\mu_{0,K}$ ) and their associated uncertainty. The difference between these predicted values and the observed values is a

measure of the offset. The standard deviation of this distribution of differences divided by  $\sqrt{N}$  gives the uncertainty on the mean of the offset. With  $C = 0$  or  $1$  the low- and high-inclination data sets are consistent (at the  $1\sigma$  level) with being parallel to each other – a requirement of the Working–Hotelling technique. When  $C = 0$  the mean vertical offset is  $-0.97 \pm 0.11 \text{ mag arcsec}^{-2}$ , and when  $C = 1$  the mean offset is  $+0.10 \pm 0.10 \text{ mag arcsec}^{-2}$ .<sup>3</sup>

The vertical scatter about the  $\mu_{0,K}$ – $\log h$  relation for the high-inclination sample is reduced by 20 per cent when the  $C = 1$  inclination correction is applied. Further support for the validity of the inclination correction comes from the improved agreement in the slope of the  $\mu_{0,K}$ – $\log h$  relation between the low- and high-inclination data sets. For the low-inclination spirals, an ordinary least-squares regression of  $\mu_{0,K}$  on  $\log h$  reveals a slope of  $2.05 \pm 0.23$  ( $C = 0$ ) and  $1.98 \pm 0.22$  ( $C = 1$ ). For the high-inclination spirals, the slope is  $1.76 \pm 0.59$  ( $C = 0$ ) and  $2.12 \pm 0.41$  ( $C = 1$ ). The error estimates for the slopes are slightly larger ( $\sim 5$ – $10$  per cent) than those obtained from the standard analytical solution which, for small sample sizes ( $N \leq 50$ ), can underestimate the size of the error. The errors shown here are the largest standard deviations obtained from jackknife and bootstrap resampling of the data. For the combined data sets, with  $C = 1$ , the mean slope is  $1.96 \pm 0.18$  with a scatter of  $0.47 \text{ mag arcsec}^{-2}$  about the  $\mu_{0,K}$ – $\log h$  relation.

With  $C = 0.91$ , the mean offset between the two samples was 0.00, but this did not result in a significantly different slope or scatter to the  $\mu_{0,K}$ – $\log h$  relation obtained when  $C = 1$ . For the low-inclination sample, with  $C = 0.91$ , the slope was  $1.99 \pm 0.21$  with a scatter of  $0.47 \text{ mag arcsec}^{-2}$ . For the high-inclination sample the slope was  $2.08 \pm 0.48$  with a scatter of  $0.49 \text{ mag arcsec}^{-2}$ , and for the combined sample the slope was  $2.01 \pm 0.18$  with a scatter of  $0.47 \text{ mag arcsec}^{-2}$  and a Pearson correlation coefficient of 0.76. Completely independent of the two inclination bins ( $i < 50^\circ$  or  $i > 50^\circ$ ), the scatter about the  $\mu_{0,K}$ – $\log h$  relation reduced to a minimum as the value of  $C$  increased to 0.9–1.0.

If one wishes to use the  $K$ -band  $\mu_{0,K}$ – $\log h$  relation for distance estimates, then one should perform a linear regression which minimizes the distance dependent quantity  $\log h$  (kpc) against the distance independent quantity  $\mu_{0,K}$ .<sup>4</sup> Performing this, with  $C = 1$ , the resulting standard deviation is a somewhat large 0.18 dex in  $\log h$  which corresponds to a mean distance uncertainty of  $\sim 50$  per cent.

#### 4.2.1 Subsampling the data

The above analysis was repeated several times, culling each data set to make them more similar to each other – and testing if these steps caused any significant changes (at the  $1\sigma$  level). First, the Sc galaxies included in the low-inclination sample were removed. This had no significant effect on any of the above numbers. Next, because the low-inclination sample has only two S0 galaxies (both with type  $T = 0$ ) while the high-inclination sample includes S0 galaxy types from  $T = -3$  to 0, all S0 and Sc galaxies were

<sup>3</sup>The Working–Hotelling confidence bands allow one to obtain two estimates of the offset by using in turn both data sets as the ‘calibration’ data set. The mean offset was not significantly different when this was done.

<sup>4</sup>Strictly speaking, the surface brightness term is dependent on the redshift via the  $K$ -correction and redshift dimming, but these corrections, or more importantly, realistic errors in these corrections as a result of errors in the redshift are less than  $0.01 \text{ mag arcsec}^{-2}$ .

removed from both samples, leaving only types  $T = 1$  to 4. Again, the numbers obtained were not significantly different from those obtained using the full sample. With  $C = 1$  the slope of the  $\mu_{0,K}$ - $\log h$  relation changed to  $2.03 \pm 0.24$  with a slightly greater Pearson correlation coefficient of 0.77.

The high-inclination galaxy sample of APB95 excluded galaxies with a prominent bar – of the 29 galaxies used from this sample, NED still lists eight of these having a bar. Therefore, the above tests were repeated excluding those galaxies from the 59 low-inclination galaxies which de Jong (1996a) considered as having a prominent bar – which he modelled with an additional bar component. None of the results differed at more than the  $1\sigma$  significance level and usually well under this. Excluding the prominently barred galaxies, the mean vertical offset between the  $\mu_{0,K}$ - $\log h$  relation for the low- and high-inclination samples is  $-1.00 \pm 0.11$  ( $C = 0$ ) and  $+0.07 \pm 0.10$  ( $C = 1$ ). The mean slope of the  $\mu_{0,K}$ - $\log h$  relation for the combined data sets changed to  $1.83 \pm 0.21$  ( $C = 1$ ) with a scatter of  $0.49 \text{ mag arcsec}^{-2}$ . Next, the eight galaxies from the high-inclination sample listed as having bars were removed as was every galaxy in the low-inclination sample which was listed in NED as having a bar. Once again, the above results did not significantly differ, and it can be concluded that these results are not biased by the presence of a bar in some of the sample members.

APB95 further restricted their sample to galaxies listed in the UGC with  $B_T < 14.0$  mag. This same restriction was applied here to the low-inclination sample to test if this may be influencing the results – it does not. Lastly, the above criteria were all applied at once to see if they may be working together to bias the above results. All S0 and Sc galaxies were excluded, as were all galaxies listed in the UGC with  $B_T < 14.0$  mag, and all galaxies considered to have a prominent bar. Ignoring the dependence of  $\mu_{0,K}$  on  $\log h$ , the mean value of  $\mu_{0,K}$  for the low-inclination sample was  $17.43 \pm 0.71$  ( $C = 0$ ) and  $17.67 \pm 0.66$  ( $C = 1$ ). For the high-inclination sample these values were  $16.05 \pm 0.82$  ( $C = 0$ ) and  $17.42 \pm 0.75$  ( $C = 1$ ). This heavy sample truncation had the greatest effect on the  $\mu_{0,K}$ - $\log h$  relation, whose slope became  $1.80 \pm 0.34$  ( $C = 1$ ) – still, however, consistent with the solution obtained with the larger galaxy sample. The scatter about the  $\mu_{0,K}$ - $\log h$  relation changed to  $0.87 \text{ mag arcsec}^{-2}$  ( $C = 0$ ) and  $0.51 \text{ mag arcsec}^{-2}$  ( $C = 1$ ).

To continue in this vein, and address constructive criticism, next, those galaxies at the tails of the scalelength distribution were excluded. Removing the one galaxy with a disc scalelength less than 1 kpc from the high-inclination sample changed the slope of the  $\mu_{0,K}$ - $\log h$  relation for the high-inclination sample to  $2.00 \pm 0.59$  ( $C = 1$ ). Removing the one galaxy from the low-inclination sample with a disc scalelength less than 1 kpc had practically no effect at all. At the other end of the distribution, five galaxies from the low-inclination sample have scalelengths larger than 10 kpc ( $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ). Although, when using the same Hubble constant as de Jong ( $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ), only two galaxies have disc scalelengths greater than 9 kpc, and only one galaxy has a disc scalelength greater than 10 kpc. Still, these five galaxies were excluded to test if they may be biasing the  $\mu_{0,K}$ - $\log h$  correlation. Doing this, the slope of the combined data set was reduced to  $1.67 \pm 0.21$  ( $C = 1$ ). Excluding these five galaxies, and those two galaxies with disc scalelengths less than 1 kpc, the slope of the combined data set became  $1.55 \pm 0.23$  ( $C = 1$ ), with a Pearson correlation coefficient of 0.60. This is still a highly significant result. The Spearman rank-order correlation coefficient is 0.58 – with a significance of  $2 \times 10^{-8}$ . Similarly, the sum-squared

difference of ranks deviates from the expectation value in the null hypothesis of uncorrelated data sets by more than five standard deviations.

As a final check, the high-inclination sample had previously been independently modelled by Khosroshahi, Wadadejar & Kembhavi (2000) using a two-dimensional image decomposition technique that fitted a seeing-convolved Sérsic bulge and exponential disc. Applying the inclination correction to the data in their table 1, and various other surface brightness and Virgo-infall corrections mentioned earlier, the above analysis was repeated again and found to be fully consistent with the results presented above.

## 5 POSSIBLE SCALELENGTH CHANGES WITH INCLINATION

For a galaxy with a globally uniform disc opacity, the surface brightness profile will brighten with inclination (or remain the same if it is optically thick) by the same amount at all radii. The scalelength of the disc would therefore be independent of inclination. If, however, the opacity of the disc changes with radius, then the scalelength could change as the galaxy was viewed at different inclination angles. This scenario opens the door to the possibility that the inclined galaxies in Fig. 3 may not have simply moved along lines of constant scalelength. In the extreme, one may postulate that discs are optically thick at their centres and  $\mu_{0,K}$  does not change with inclination, but instead the scalelength changes to produce the shift in Fig. 3. However, rather than acting as a hindrance to the question of disc opacity, this effect is rather fortuitous as it actually enables us to additionally constrain possible changes in the opacity with radius.

If discs are more dusty in their central parts than in their outer parts, then the disc scalelengths will increase with inclination. Indeed, there is evidence that discs may in fact be more optically thick at their centres (Bosma et al. 1992; Huizinga & van Albada 1992; Giovanelli et al. 1994; Jansen et al. 1994; Peletier et al. 1995; Beckman et al. 1996; Trewheella et al. 1997). As such galaxies are viewed with increasing inclination, the outer parts of the discs will brighten more than the inner parts; this would act to reduce the slope of the disc surface brightness profile and therefore increase the disc scalelength from its face-on value – although this effect will be less in the near-infrared than in the optical.

On average, in the  $K$ -band, the decline in opacity with radius must be fairly uniform from the disc centre to the outer boundary. If this were not the case, the surface brightness profiles of the inclined discs would be seen to systematically deviate from an exponential light distribution (having Sérsic shape parameters  $n > 1$ ), and possess kinks and gradient changes as a result of uneven surface brightness increases. Of course some profiles do possess such features, and some display a brightening of the outer disc profile above that defined by the exponential behaviour of the inner disc profile. This could be a signature of a disc which is transparent only in the outer parts and is brightening with inclination. However, such galaxies in the sample which behave this way have no preference for higher inclinations, and the reason for such gradient changes therefore seems likely to be because of other causes, perhaps resulting from an underestimation of the sky-background level. The overwhelming majority of  $K$ -band disc profiles in the sample are well described by an exponential law.

How realistic can changes to the scalelength be for a uniform decrease in the opacity with radius? Obviously the greatest change will occur if the centre of the disc is opaque and the outer measured

parts of the disc are fully transparent. For a galaxy observed at some random inclination, with a central disc opacity  $C$  and the outer disc fully transparent (i.e.  $C = 1$ ), one can compute the change in scalelength, as a result of inclination, from the face-on value. For a disc with a measured scalelength  $h_{\text{obs}}$  derived from its light profile data which extends to  $x$  scalelengths, the face-on scalelength is given by the expression

$$h_{\text{face-on}} = h_{\text{obs}} \frac{x}{x + \Delta\mu/1.086}, \quad (2)$$

where  $\Delta\mu = (1 - C) \times 2.5 \log(a/b)$ . For the high-inclination sample of disc galaxies, the average maximum reduction to the disc scalelength which can be achieved assuming radial differences in the transparency of the disc (with  $C = 1$  at the outer radius, and  $C = 0$  at the inner radius) is only  $23 \pm 6$  per cent. This is not enough to bring the low- and high-inclination galaxies into agreement in Fig. 3; in fact, it is short by about a factor of 10. The disc scalelengths of the low-inclination galaxies undergo an average reduction of  $6 \pm 4$  per cent when corrected, under the previous assumption, to their face-on value.

Any surface brightness and scalelength inclination correction to the face-on value will of course affect the high-inclination sample the most. It is therefore of interest to see how the scatter about the  $\mu_{0,K}$ – $\log h$  relation changes in this sample when both of these inclination corrections are applied. Changing the central value of  $C$  and correcting  $\mu_{0,K}$  and  $\log h$  using equations (1) and (2) (i.e. assuming the outer disc is fully transparent), an ordinary least-squares bisector line of regression was fitted to the data. A bisector line of regression treats both variables equally, as opposed to the ordinary least-squares regression of  $\mu_{0,K}$  on  $\log h$  which was used in the previous section where the scatter was considered only to arise from the surface brightness term (see Feigelson & Babu 1992 for a discussion). The perpendicular scatter, and also the vertical scatter in  $\mu_{0,K}$ , were observed to steadily decrease to a minimum as the central disc value for  $C$  approached one. This was also the case when an orthogonal regression line was used. When fitting a bisector regression line to the combined low- and high-inclination galaxy sample, the minimum perpendicular scatter, and also vertical scatter in  $\mu_{0,K}$ , were achieved when  $C = 0.9$  – a result independent of the two inclination bins.

To explore this further, a bisector line of regression was individually fitted to each of the two data samples and the Working–Hotelling method was adapted to compute the central value of  $C$  which gives the smallest perpendicular offset between the two samples in the  $\mu_{0,K}$ – $\log h$  plane. Correcting  $\mu_{0,K}$  and changing the scalelength assuming the maximum possible opacity gradient, when using the 59 low-inclination galaxies as the calibration set in the Working–Hotelling method, the minimum offset occurred when  $C = 0.89$ . Using the 29 high-inclination galaxies as the calibration set, this occurred when  $C = 0.97$ .

To check for differences arising between the ordinary least-squares regression of  $\mu_{0,K}$  on  $\log h$  and the bisector method of regression, the latter was again applied to the data while the scalelengths were held constant, and only corrections to the surface brightness applied – as was done previously. The minimum perpendicular offset between the low- and high-inclination galaxies was found to occur when  $C = 0.91$  and  $0.98$  when the low- and high-inclination galaxies were respectively used as the calibration data set.

## 6 DISCUSSION

One of the problems, and hence some of the uncertainty today, with

some of the past surface brightness inclination tests was that they used the ‘mean’ surface brightness ( $\langle\mu\rangle$ ) within some fixed isophotal radius. Consequently, one did not know if changes to the mean surface brightness, with inclination, were because of the changes to the enclosed integrated magnitude or because of changes to the isophotal radius (Chołonewski 1991; Burstein, Haynes & Faber 1991; Huizinga & van Albada 1992). With central surface brightnesses ( $\mu_0$ ) this is not a problem. Furthermore, the selection criteria biases from either a magnitude-limited or diameter-limited sample, which have previously been illustrated to operate in the apparent magnitude–apparent diameter ( $B_T$ – $\log D$ ) diagram (Burstein et al. 1991), do not apply to the  $\mu_0$ – $\log D$  or  $\mu_0$ – $\log h$  diagram. For example, although the use of apparent magnitudes from a diameter-limited sample of galaxies would bias one to conclude that discs are optically thick (Burstein et al. 1991), this is not the case when one is using central surface brightnesses – as observed in this investigation.

Increasing the inclination of optically thin discs from face-on, their isophotal radii increase while their magnitudes remain constant; if they are optically thick, their magnitudes decrease while their isophotal radii remain constant. Independent of the opacity of the disc, this situation results in ‘inclined’ galaxies changing positions in the  $B_T$ – $\log D$  plane from their face-on values, and leaving one uncertain as to whether the apparent magnitude, the isophotal diameter or a combination of both has changed. Burstein et al. (1991) and Chołonewski (1991) tried to resolve this dilemma by using distance information to the galaxies, but Davies et al. (1995) showed the presence of the redshift-cut-off selection criteria left this mean-surface-brightness inclination test unable to tell one anything about the transparency of discs. This dilemma, however, does not exist in the  $\mu_0$ – $\log D$  diagram. If discs are optically thick, then neither  $\mu_0$  nor  $\log D$  will change with inclination. If discs are optically thin then they will move towards brighter  $\mu_0$  and larger  $\log D$  values as they are viewed at greater inclination angles. Therefore, any positional correlation with inclination in the  $\mu_0$ – $\log D$  diagram will indicate that discs are at least semi-transparent; a lack of any such trend will indicate that they are opaque. The strength of any such trend depends on the inclination of the sample (which is known) and the apparent opacity of the disc (which one could then measure). The situation is even simpler here because scalelengths, rather than isophotal diameters, have been used, and for a globally constant disc opacity the former will not change with inclination – although possible radial changes in the opacity of the disc have been explored here.

For transparent galaxies, as the inclination angle increases the isophotal diameters will increase. Consequently, the high-inclination galaxy sample may contain galaxies which are intrinsically smaller than the galaxies in the low-inclination sample. Although, as noted by Huizinga (1994), this may not be such a problem for galaxies selected from the UGC. Their diameters were estimated by eye to the faintest structures visible on the photographic plates. Given that these structures are probably H II regions, whose surface brightness does not change with inclination, the diameter inclination dependence is probably small. Anyway, plotting the central surface brightness against disc scalelength should account for any selection bias owing to galaxy size in this inclination test.

This investigation is free from criticism levelled at previous studies which used central disc surface brightnesses without accurately considering the contribution of the bulge light to the disc profile. Kormendy (1977) and Davies (1990) have shown how the linear part of the surface brightness profile can be contaminated



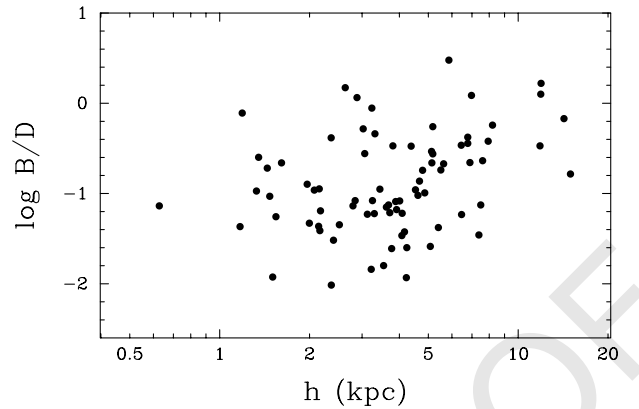
by bulge light, and how fitting to this part of the profile alone can result in erroneous estimates of the central disc surface brightness. In the analysis presented here, Sérsic  $r^{1/n}$  models convolved with a Gaussian point spread function (PSF) have been fitted to the bulge while an exponential model, also convolved with the PSF, was simultaneously fitted to the disc. For the low-inclination sample, the full width at half-maximum (FWHM) of the PSF had been individually obtained for each galaxy from stars on the image frames (de Jong 1996a). For the high-inclination sample, a value of 1.0 arcsec was used – the actual seeing varied from 0.8 to 1.2 arcsec but this had little effect on the results of the disc parameters (see also Khosroshahi et al. 2000).

There have been a few near-infrared investigations, employing different techniques, to measure the value of  $C$  required for the inclination correction. Using 1.65- $\mu\text{m}$  ( $H$ -band) aperture photometry, Peletier & Willner (1992) and Boselli & Gavazzi (1994) presented evidence that discs behaved in a semitransparent way: with  $C_H \geq 0.65$  and  $C_H = 0.5\text{--}0.9$ , respectively. A more recent study using a two-dimensional bulge/disc decomposition found  $C_H = 0.60 \pm 0.14$  (Moriondo et al. 1998). With a  $K'$  study, Tully & Verheijen (1997) found that the minimum scatter in the central disc surface brightness for their high surface brightness (HSB) galaxies occurred when  $C_{K'} = 1$ , in agreement with the  $K$ -band analysis of this paper. Given that near-infrared (2.2- $\mu\text{m}$ ) light passes largely undisturbed through dust clouds, the line-of-sight depth into (and through?) a spiral galaxy disc is expected to increase as one views spiral galaxies, and hence discs, at greater inclinations. It is therefore reassuring to find that the observed central disc surface brightness does increase with galaxy inclination in the  $K$ -band.

The decision to use spiral galaxies with morphological types  $T \leq 5$  was determined by two factors. First, the high-inclination galaxy samples were all of this type. It should perhaps be mentioned that difficulties in matching morphological types at different inclinations may introduce some small bias, but given the inherent uncertainty in this business (Lahav et al. 1995) this is not expected to be important for this sample. The other reason for using the early-type galaxies is that the late-type spirals do not follow the  $\mu_{0,K}\text{--}\log h$  relation evident for the early-type disc galaxies. For a given scalelength, galaxy types later than Sc can be considerably fainter than the earlier-type spirals. To show this, Fig. 3 displays the data for the low-inclination disc galaxies having type  $T > 5$ . This data has not been included in any of the calculations presented previously.

Further evidence to segregate the late-type and early-type disc galaxies comes from the work by Tully & Verheijen (1997) who showed that the S0–Sc and Scd–Sm–Irr galaxies have markedly different central surface brightness values, not just as a function of Hubble type but also in the  $\mu_0\text{--}\log h$  plane (but see Bell & de Blok 2000). Considering Fig. 3 from this paper with fig. 2 from van den Bosch (1998) – which shows a V-band  $\mu_0\text{--}\log h$  relationship for all types of galaxy discs – one can see that the late-type spirals from the sample of de Jong & van der Kruit (1994) reside in the same part of the diagram as the LSB galaxies. Previous inclination tests which have looked for a correlation between  $\mu_0$  and  $\log(b/a)$  may therefore suffer confusion/noise because of the inclusion of late-type galaxies ( $T > 5$ ) and because of the relationship between surface brightness and scalelength amongst the early-type spirals.

Could selection effects be responsible for this trend between  $\mu_{0,K}$  and  $\log h$ , by preventing early-type galaxies from occupying the upper-right or lower-left of the  $\mu_{0,K}\text{--}\log h$  diagram? The selection criteria are such that they favour the detection of galaxies with large scalelengths and luminous central surface brightnesses. At



**Figure 4.** Bulge-to-disc ratio versus disc scalelength for galaxies with morphological type  $\leq \text{Sc}$ . The full  $C = 1$  inclination correction has been applied to the disc (see text for details). A Hubble constant of  $75 \text{ km s}^{-1} \text{ Mpc}^{-1}$  has been used.

a fixed scalelength, the probability of detecting galaxies that have central surface brightnesses, which are brighter than those observed along the  $\mu_{0,K}\text{--}\log h$  relation, is actually greater than the probability of detecting those galaxies at the same scalelength which define the observed relation. Therefore, the upper envelope in the  $\mu_{0,K}\text{--}\log h$  diagram reflects a true drop-off in the number density of bigger, brighter spiral galaxies, and has been noted before.

Grosbøl (1985) suggested that it might be because of an upper constant luminosity limit, in which case the slope should be five. However, his photographic data appears more consistent with a slope of  $\sim 3$ . Fig. 4 shows that for the early-type spiral galaxies, the  $B/D$  luminosity ratio is, if anything, smaller for the smaller scalelength galaxies. Therefore, the  $\mu_{0,K}\text{--}\log h$  relation clearly does not delineate a constant upper luminosity boundary to the luminosity function – even when the presence of the bulge light is factored in. Fitting a line of constant luminosity in the  $\mu_0\text{--}\log h$  diagram with a slope of 2.5 instead of 5, Kent (1985) mistakenly attributed the  $\mu_0\text{--}\log h$  correlation to a limited range of disc luminosities in his sample. Using the  $I$ -band Sb–Sd galaxy data from Byun’s (1992) thesis, the slope of this envelope is  $\sim 1.9$  (see also Reshetnikov 2000), and a somewhat smaller value of 1.5 is obtained from the composite data of Dalcanton, Spergel & Summers (1997, their fig. 4). Evstigneeva & Reshetnikov (1999) have claimed, from a compilation of 356 galaxies, that normal HSB spiral discs follow a trend such that  $\mu_0 \propto 2.5 \log h$  with a correlation coefficient of 0.45 and a scatter of  $\sim 1 \text{ mag arcsec}^{-2}$ . After submission, the author discovered that Möllenhoff & Heidt (2001) reported a slope of  $2.02 \pm 0.64$  ( $r = 0.61$ ) in the near-infrared for a sample of 40 bright, low-inclination, early-type ( $\leq \text{Sc}$ ) spiral galaxies without bars, in complete agreement with the results obtained here (see also Seigar & James 1998, Moriondo, Giovanardi, & Hunt 1998 and Khosroshahi et al. 2000).

Recent studies of bulge dominated LSB galaxies (Beijersbergen, de Blok & van der Hulst 1999) have shown that they appear to continue the trend defined by the normal early-type HSB galaxies which have been used here. With considerably more scatter still, a similar trend exists not just for the ordinary HSB spiral galaxy discs, but also for the LSB discs, discy ellipticals and HST resolved nuclear discs (Scorza & van den Bosch 1998; Reshetnikov 2000). For a constant velocity, the virial theorem, which can be expressed as  $v^2 \propto I/h$ , predicts a slope of 2.5 in the  $\mu_0\text{--}\log h$  diagram. How, and whether or not, this could be the limiting criteria is not clear.



The location of the late-type spirals in the lower-left of the  $\mu_{0,K} - \log h$  diagram, in the region of lower rotational velocity, is in accord with observational results (Zaritsky, 1993, his fig. 2). The selection criteria do however somewhat restrict the galaxy sample from occupying this part of the diagram. The dotted lines in the upper panel of Fig. 3 reveal the nature of the selection criteria and show the completeness of the sample to the indicated distances (15, 40, 100 Mpc;  $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ). Assuming perfectly exponential discs, these lines show the 2.0-arcmin diameter limit at the  $22.45 \text{ K-mag arcsec}^{-2}$  isophote. (The average surface brightness where the UGC red diameters were measured is  $24.7 \text{ R-mag arcsec}^{-2}$ , and the mean  $R - K$  colour at the 60-arcsec radius for those galaxies in the low-inclination sample with photometric data and morphological type  $1 \leq T \leq 5$  is  $2.25 \pm 0.07$ .) Galaxies near the 100-Mpc line had  $(100/15)^3 \sim 300$  times more chance of being included in the sample than those near the 15-Mpc line. For any  $(\mu_{0,K}, \log h)$  parameter combination between two close pairs of these curved lines, there is an equal probability of observing a galaxy if it exists in, at least, the local Universe. Therefore, within any such curved region, the absence of early-type disc galaxies below the  $\mu_{0,K} - \log h$  relation indicates that they do not exist here in as great a number density as the population on the observed  $\mu_{0,K} - \log h$  relation and within the same curved bands. The presence of the late-type galaxy population in the lower part of the  $\mu_{0,K} - \log h$  diagram suggests that these galaxies exist with a greater number density than the early-type spirals in this domain of the parameter space.<sup>5</sup> The inclusion of disc-dominated LSB galaxies is known, at least at shorter wavelengths, to fill in the lower left of the diagram (Bothun et al. 1997, their fig. 4; Dalcanton et al. 1997; McGaugh 1998). Whether or not, in the  $K$ -band, early-type spiral galaxies reside in the lower left of the  $\mu_0 - \log h$  diagram in any great number density cannot be answered with the present data set. Consequently, while the observed  $\mu_0 - \log h$  relation may reflect the location to the bulk of the early-type galaxy population, it may also only reflect the upper envelope – which is, in itself, still of interest. It would be of benefit to extend the current data sample to smaller discs and fainter surface brightness levels.

## 7 MAIN CONCLUSIONS

The well-known modification to Freeman's law to incorporate fainter central disc surface brightnesses is, in some respect, partially quantified: a relation is presented for the discs of early-type spiral galaxies. Their central disc surface brightnesses, at least in the  $K$ -band, are shown to increase (that is, become fainter) with increasing disc scalelength, such that  $\mu_{0,K} \propto 2 \log h$ . This relation is not adhered to by the late-type ( $\geq \text{Scd}$ ) spiral galaxies.

It is found that the discs of early-type spiral galaxies ( $\leq \text{Sc}$ ) should be considered transparent for the sake of  $K$ -band inclination corrections to their observed photometric parameters. From a number of tests, the optimal value of  $C$  in the standard inclination correction formula of  $2.5C \log(b/a)$ , where  $b/a$  is the minor-to-major-axis ratio, resides between 0.9 and 1.0. This implies that the maximum correction, in the confines of the inclination correction formula, should be applied to the observed  $K$ -band disc surface brightness, and simultaneously, no corrective term for dust should be applied to the observed  $K$ -band magnitudes.

<sup>5</sup>The late-type galaxies are some  $0.30 \pm 0.16 \text{ mag}$  bluer in  $R - K$  than the early-type galaxies, and so the selection criteria lines shown in Fig. 3 for the early-type spirals will be different by this amount for the late-type spirals.

## ACKNOWLEDGMENTS

I am grateful to John Beckman and Marc Balcells for their reading and comments on this paper. I also thank Peter Erwin for enjoyable discussions, Eric Feigelson for making the computer code SLOPES and CALIB publicly available and Renée Kraan-Korteweg for sending me her Virgo-centric inflow code. I thank Gregory Bothun and R. de Jong for their input and suggestions which helped to improve this paper. This research has made use of the NASA/IPAC Extragalactic Database (NED) which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

## REFERENCES

- Andersen D. R., Bershadsky M. A., Sparke L. S., Gallagher J. S., Wilcots E. M., Van Driel W., Ragaigine D., 2000, in Funes J. G., SJ, Corsini E. M., eds, ASP Conf. Ser. Vol. 230, Galaxy Disks and Disk Galaxies. Astron. Soc. Pac., San Francisco, in press
- Andredakis Y. C., Peletier R. F., Balcells M., 1995, MNRAS, 275, 874 (APB95)
- Aoki T., Hiromoto N., Takami H., Okamura S., 1991, PASJ, 43, 755
- Balcells M., Peletier R. F., 1994, AJ, 107, 135
- Barnaby D., Thronson H., 1992, AJ, 103, 41
- Beckman J. E., Peletier R. F., Knapen J. H., Corradi R. L. M., Gentet L. J., 1996, ApJ, 467, 175
- Beijersbergen M., de Blok W. J. G., van der Hulst J. M., 1999, A&A, 351, 903
- Bell E. F., de Blok W. J. G., 2000, MNRAS, 311, 668
- Borson T., 1981, ApJS, 46, 177
- Borson T., 1983, ApJ, 274, 39
- Boselli A., Gavazzi G., 1994, A&A, 283, 12
- Bosma A., Byun Y., Freeman K. C., Athanassoula E., 1992, ApJ, 400, L21
- Bothun G., Impey C., Malin D., Mould J., 1987, AJ, 94, 23
- Bothun G., Impey C., McGaugh S., 1997, PASP, 109, 745
- Burstein D., 1979, ApJ, 234, 435
- Burstein D., Haynes M. P., Faber S. M., 1991, Nat, 353, 515
- Byun Y.-I., 1992, PhD thesis, Univ. Australia
- Choloniewski J., 1991, MNRAS, 250, 486
- Dalcanton J. J., Spergel D. N., Summers F. J., 1997, ApJ, 482, 659
- Davies J. I., 1990, MNRAS, 244, 8
- Davies J. I., Phillipps S., Disney M. J., 1988, MNRAS, 231, 69p
- Davies J. I., Phillipps S., Boyce P. J., Disney M. J., 1993, MNRAS, 260, 491
- Davies J. I., Jones H., Trewhella M., 1995, MNRAS, 273, 699
- de Blok W. J. G., van der Hulst J. M., Bothun G. D., 1995, MNRAS, 274, 235
- de Blok W. J. G., van der Hulst T., McGaugh S., 1996, Bull. Am. Astron. Soc., 189, 84.02
- de Grijs R., 1998, MNRAS, 299, 595
- de Jong R. S., 1996a, A&AS, 118, 557
- de Jong R. S., 1996b, A&A, 313, 45
- de Jong R. S., van der Kruit P. C., 1994, A&AS, 106, 451
- de Vaucouleurs G., 1948, Ann. d'Ap, 11, 247
- de Vaucouleurs G., de Vaucouleurs A., Corwin H. G., Jr., Buta R. J., Paturel G., Fouque P., 1991, Third Reference Catalog of Bright Galaxies. Springer-Verlag, New York (RC3)
- Disney M. J., 1976, Nat., 263, 573
- Disney M. J., 1980, in Tarenghi M., Kiar T. K., eds, Dwarf Galaxies. ESO/ESA Workshop. ESO/ESA, Geneva, p. 151
- Disney M. J., Davies J., Phillipps S., 1989, MNRAS, 239, 939
- Evstigneeva E. A., Reshetnikov V. P., 1999, Astron. Lett., 25, 582
- Feigelson E. D., Babu G. J., 1992, ApJ, 397, 55
- Freeman K. C., 1970, ApJ, 160, 811
- Giovannelli R., Haynes M. P., Salzer J. J., Wegner G., Costa L. N., Freudling W., 1994, AJ, 107, 2036

- Graham A. W., 2000a, in Vilchez J. M., Stasinka G., Perez E. eds, *The Evolution of Galaxies. I – Observational Clues*, *Astroph. and Space Sci.*, Vol. 277. Kluwer, Dordrecht, in press
- Graham A. W., 2000b, in Corsini E. M., Funes J. G., SJ, eds, *ASP Conf. Ser. Vol. 230, Galaxy Disks and Disk Galaxies*. Astron. Soc. Pac., San Francisco, in press
- Graham A. W., 2001, *AJ*, 121, 820
- Graham A. W., Prieto M., 1999, *ApJ*, 524, L23
- Grosbøl P. J., 1985, *A&AS*, 60, 261
- Huizinga J. E., 1994, PhD thesis, Univ. Groningen, the Netherlands
- Huizinga J. E., van Albada T. S., 1992, *MNRAS*, 254, 677
- Jansen R. A., Knapen J. H., Beckman J. E., Peletier R. F., Hes R., 1994, *MNRAS*, 270, 373
- Kent S., 1985, *ApJS*, 59, 115
- Khosroshahi H. G., Wadadekar Y., Kembhavi A., 2000, *ApJ*, 533, 162
- Kormendy J., 1977, *ApJ*, 217, 406
- Kraan-Korteweg R. C., 1986, *A&AS*, 66, 255
- Lahav O. et al., 1995, *Sci*, 267, 859
- Leroy P., Portilla M., 1998, *A&A*, 329, 840
- Longmore A. J., Hawarden T. G., Goss W. M., Mebold U., Webster B. L., 1982, *MNRAS*, 200, 325
- McGaugh S., 1996, *MNRAS*, 280, 337
- McGaugh S., 1998, in Davies J. I., Impey C., Phillipps S., eds, *The Low Surface Brightness Universe*, *Proc. IAU Colloq. 171*, *ASP Conf. Ser. Vol. 170, The Low Surface Brightness Universe*. Astron. Soc. Pac., San Francisco, p. 19
- McGaugh S., Bothun G. D., 1994, *AJ*, 107, 530
- Möllenhoff C., Heidt J., 2001, *A&A*, 368, 16
- Moriondo G., Giovanardi C., Hunt L. K., 1998, *A&AS*, 130, 81
- Moriondo G., Giovanelli R., Haynes M. P., 1998, *A&A*, 338, 795
- Nilson P., 1973, *Uppsala General Catalog of Galaxies (Roy. Soc. Sci., Uppsala) (UGC)*
- Peletier R. F., Balcells M., 1997, *New Astronomy*, 1, 349
- Peletier R. F., Willner S. P., 1992, *AJ*, 103, 1761
- Peletier R. F., Valentijn E. A., Moorwood A. F. M., Freudling W., 1994, *A&AS*, 108, 621
- Peletier R. F., Valentijn E. A., Moorwood A. F. M., Freudling W., Knapen J. H., Beckman J. E., 1995, *A&A*, 300, L1
- Phillipps S., Disney M. J., 1983, *MNRAS*, 203, 55
- Poggianti B. M., 1997, *A&AS*, 122, 399
- Reshetnikov V. P., 2000, *Astron. Lett.*, 26, 485
- Schlegel D. J., Finkbeiner D. P., Davis M., 1998, *ApJ*, 500, 525
- Schombert J. M., Bothun G. D., 1987, *AJ*, 92, 60
- Scorza C., van den Bosch F. C., 1998, *MNRAS*, 300, 469
- Seigar M. S., James P. A., 1998, *MNRAS*, 299, 672
- Sérsic J.-L., 1968, *Atlas de Galaxias Australes*. Observatorio Astronomico, Cordoba
- Trewhella M., Davies J. I., Disney M. J., Jones H. G. W., 1997, *MNRAS*, 288, 397
- Tully R. B., Verheijen M. A. W., 1997, *ApJ*, 484, 145
- Valentijn E. A., 1990, *Nat*, 346, 153
- van den Bosch F. C., 1998, *ApJ*, 507, 601
- van der Hulst J. M., Skillman E. D., Smith T. R., Bothun G. D., McGaugh S. S., de Blok W. J. G., 1993, *AJ*, 106, 548
- van der Kruit P. C., 1987, *A&A*, 173, 59
- Working H., Hotelling H., 1929, *J. Am. Stat. Assoc. Suppl.*, 24, 73
- Zaritsky D., 1993, *PASP*, 105, 1006

This paper has been typeset from a  $\text{\LaTeX}$  file prepared by the author.